Evaluation of cryo-hydrologic warming as an explanation for increased ice velocities in the wet snow zone, Sermeq Avannarleq, West Greenland

Thomas Phillips, Harihar Rajaram, William Colgan, Konrad Steffen, and Waleed Abdalati

Received 31 July 2012; revised 16 November 2012; accepted 14 March 2013.

[1] Wintertime satellite-derived ice surface velocities, from 2001 through 2007, suggest an increase in ice velocity in the wet snow zone of Southwest Greenland. We present a thermomechanical model to evaluate the influence of surface meltwater runoff on englacial temperatures, via cryo-hydrologic warming (CHW), as a possible mechanism to explain this velocity increase at Sermeq Avannarleq. The model incorporates CHW through a previously published dual-column parameterization. We compare model simulations with (i) CHW active over the entire ice thickness (“base case CHW”), (ii) CHW active only in the surface 80 m of the ice sheet (“surface CHW”), and (iii) “no CHW” to represent a traditional thermomechanical model. The horizontal extent of CHW is prescribed based on equilibrium line altitude position and thus incorporates the upstream expansion of the ablation zone over the past decade. The base case CHW simulations reproduce the observed increase in inland ice velocity between 2001 and 2007 reasonably well. The no CHW and surface CHW simulations significantly underestimate observed ice surface velocities in both epochs. The higher ice velocities in the base case CHW simulations are attributable to both decreased basal ice viscosities associated with increased basal ice temperatures and an increase in the extent of basal sliding permitted by temperate bed conditions. Only the temperate bed extent predicted by the base case CHW simulation is consistent with independent observations of basal sliding. Based on our sensitivity analysis of CHW, we evaluate alternative explanations for an increase in inland ice velocity and suggest CHW is the most plausible mechanism.


1. Introduction

[2] Meltwater is generated in the ablation, wet snow, and percolation zones of glaciers and ice sheets. Within the lower accumulation zone, in the wet snow and percolation zones, a fraction of this meltwater refreezes within the upper layers of the firm, where it acts as a near-surface latent heat source [Benson, 1961; Hooke, 1976]. The remaining fraction of meltwater enters the englacial hydrologic system via crevasses and/or moulins and is routed to the subglacial hydrologic system at the bed of the glacier or ice sheet [Fountain and Walder, 1998]. The potential warming of ice by meltwater contained in the englacial and subglacial hydrologic systems has been previously observed and modeled [Bader and Small, 1955; Jarvis and Clarke, 1974; Phillips et al., 2010].

[3] In 1953, the U.S. Air Force established inhabited radar stations at high elevations of the Greenland Ice Sheet. These stations discharged approximately $2.1 \times 10^6 \text{ L}$ of 10 to $13^\circ \text{C}$ waste water per year into unlined sump pits within the firm. Despite being encased by ice colder than $–20^\circ \text{C}$, the waste water had not completely refrozen when a sump was resurveyed 2 years after its closure [Ostrom et al., 1962]. Manual inspection of the sump confirmed that the ice at depth had been “honeycombed by cavities, saturated with water, and made more plastic than its surroundings by higher temperature” [Bader and Small, 1955]. Large spatial gradients in ice deformation observed beneath the station, approximately 40 m horizontally away from the sump, were attributed to the warm ice surrounding the sump. These qualitative observations
suggested that pockets of liquid water could significantly modify the temperature, and thus deformation, of cold ice for several years.

[4] Jarvis and Clarke [1974] observed significantly elevated temperatures extending over a 10 to 100 m depth range on Steele Glacier, a cold-based glacier in Yukon Territory, Canada. The warmest ice temperatures, within 1°C of the pressure melting point, were approximately 7°C greater than the ice temperatures expected based on the geothermal gradient and mean annual air temperature. Thermodynamic modeling suggested that these elevated temperatures were due to the presence of a nearby crevasse that had filled with meltwater during a surge 7 years earlier and had not yet completely refrozen. Clarke and Jarvis [1976] suggested that liquid water could persist without refreezing in these crevasses for two to three decades. These observations and modeling results confirmed the ability of a single meltwater pulse to significantly elevate ice temperature for several years in a natural setting.

[5] To further understand the warming of ice by heat transferred from water-filled crevasses, let us consider a simple thought experiment involving an array of regularly spaced long crevasses, similar to the conceptual model of Jarvis and Clarke [1974]. Field observations and satellite imagery indicate that crevasses in the vicinity of Sermeq Avannarleq, West Greenland, are several kilometers long and a few meters wide [Colgan et al., 2011b] (Figure 1). Crevasses that are many orders of magnitude longer than their width are common throughout Greenland. Beginning from an initial ice temperature of −10°C, let us consider the energy transfer from 1 m wide and infinitely long crevasses that extend through the ice and are filled by a one-time input of 0°C meltwater. For simplicity, let us consider ice depths below which there is no influence of seasonal variations in air temperature and estimate the thermodynamic interactions between water-filled crevasses and ice. The final temperature attained after thermodynamic equilibrium may be readily calculated by considering the higher enthalpy due to liquid water in 1 m width for every “2R” m width of cold ice (where 2R denotes the spacing between crevasses). The final equilibrium ice temperatures are −6.3°C for 2R = 50 m and −8.2°C for 2R = 100 m. The corresponding increase in the temperature-dependent flow law parameter (A) is a factor of 3.2 and 1.8, respectively. This doubling or tripling of the flow law parameter can have significant thermomechanical consequences on ice velocity. Of course, the complete refreezing of the water and warming of the ice do not necessarily happen within the same year as the meltwater input. The duration required for attaining thermodynamic equilibrium depends on both the crevasse width and spacing and the initial ice temperature. Previously published modeling and dimensional analysis suggests that the time scale for attaining thermodynamic equilibrium is strongly controlled by the characteristic time scale $R^2 / \kappa$, where $R$ is the half-spacing between crevasses and $\kappa$ is the thermal diffusivity of ice [Phillips et al., 2010]. In the above calculations, the approximate durations required for thermodynamic equilibrium are 4 years ($2R = 50$ m) and 16 years ($2R = 100$ m).

[6] In the above thought experiment, regularly spaced crevasses were assumed to undergo a one-time filling, which is a rather conservative assumption. In reality, the dynamics of the englacial cryo-hyrdologic system (CHS) are much more complicated [Fountain and Walder, 1998]. Field observations in the vicinity of Sermeq Avannarleq suggest the persistence of englacial liquid water for many years in both crevasse and moulin-dominated englacial systems [Catania and Neumann, 2010; McGrath et al., 2011]. The presence of diffuse englacial radar diffusers has been interpreted to suggest a persistent distributed englacial network. The classical observations of Holmlund and Hooke [1983] and Holmlund [1988] document the complex geometry and temporal evolution of the CHS. Various observations [Boon and Sharp, 2003; Alley et al. 2005; Das et al., 2008; Benn et al., 2009] document or suggest the role of fracture propagation in forming new englacial water passages in cold ice in single or multiple events, which further facilitates meltwater delivery to a constantly evolving CHS. There are also other factors
such as rate of drainage (e.g., moulins versus crevasses [Colgan et al., 2011b]) that influence water retention in the CHS. Recent work suggests that a significant fraction (~50%) of annual ice sheet surface runoff may be retained in the CHS [Rennermalm et al., 2012]. Thus, it is highly likely that liquid water is introduced to the CHS on an annual basis, providing an energy source with the potential to warm the surrounding ice. For every 1% by ice sheet volume of water retained, the ultimate ice warming potential after full refreezing is ~1.8°C.

[7] Phillips et al. [2010] demonstrated that historical ice temperature profiles observed at Sermeq Avannarleq could not be reproduced without accounting for latent heat transfer from the CHS and termed this mechanism “cryo-hydrologic warming” (CHW). Consistent with Jarvis and Clarke [1974], and the above thought experiment, Phillips et al. [2010] demonstrated that CHW could potentially lead to significant warming of ice temperatures within decades of the onset of an annual melt cycle. This interpretation of historical data suggested that CHW was active in the Sermeq Avannarleq ablation zone prior to the circa 1990 rapid increase in surface air temperatures and meltwater production. Due to the upward migration of Greenland’s snow zones, driven by increasing atmospheric temperatures, broad regions of the Greenland Ice Sheet will begin to experience meltwater inputs, as the historical percolation zone transitions to contemporary wet snow zone. In these regions, Phillips et al. [2010] suggested that CHW might facilitate a relatively rapid change in ice sheet temperature and hence deformational velocity. Their calculations suggested an ice sheet thermal response time scale of decades, as opposed to the millennial thermal response timescales estimated from conventional thermodynamic models [Johannesson et al., 1989].

[8] The area of West Greenland experiencing surface melt is increasing at a rate of approximately 3.9% per year, in response to a >200 m increase in equilibrium line altitude (ELA) between 1990 and 2010 [Ettema et al., 2009]. Within this region, Sermeq Avannarleq, the first tidewater glacier north of Jakobshavn Isbrae, provides one of the most constrained modeling targets due to an abundance of observational data [Thomsen and Thorning, 1992; Steffen and Box, 2001; Zwally et al., 2002; Joughin et al., 2010a] and previous modeling studies [Phillips et al., 2010; Colgan et al., 2011a; Colgan et al., 2012]. Surface mass balance modeling suggests that the mean ELA at Sermeq Avannarleq was slightly above 1400 m during the 1980 to 1990 period and increased to 1600 m during the 2000 to 2010 period [Ettema et al., 2009]. In response to this vertical ascent, the ELA has moved inland from 82 to 98 km upstream of the terminus. As a result, a 16 km wide band of the lower accumulation zone, the historical wet snow zone, has transitioned into contemporary ablation zone. In section 2 below, we present observations showing that the contemporary wet snow zone, the historical percolation zone, has experienced a significant increase in ice velocity. In this paper, we explore whether CHW, driven by an upward migration of the ELA, may be responsible for the increased ice velocity in the contemporary wet snow zone of Sermeq Avannarleq.

[9] The highly uncertain geometry and temporal evolution of the CHS precludes representing CHW with precise high-resolution thermodynamic models. For this reason, Phillips et al. [2010] proposed a simple parameterization in which the background ice and the CHS are viewed as overlapping continua, with heat exchange driven by the temperature difference between the two continua. Similar parameterizations are widely used for transport processes in fractured rock, where the precise topology of the fracture networks is highly uncertain [e.g., Barenblatt et al. 1960; Pruess and Narasimhan, 1985]. Phillips et al. [2010] employ an energy exchange rate parameter $\kappa / R^2$ (with units of time $^{-1}$) between background ice and the CHS, where $R$ is viewed as a characteristic half-spacing between elements of the CHS (e.g., the CHS cuts through the ice producing blocks with a characteristic dimension of 2$R$). In crevasse fields, $R$ may be viewed as the half-spacing between crevasses. Even in the case of moulin shafts and conduits, or networks thereof, the characteristic time scale for heat transfer scales as $\kappa / R^2$ albeit with corrections for dimensionality (e.g., radial heat transfer). However, the poorly constrained and complex internal geometry of the CHS complicate the detailed quantification of the energy transfer time scales. For instance, McGrath et al. [2011] observed large lateral englacial conduits and a number of fractures filled with refrozen meltwater within the background ice encasing a moulin. While we acknowledge that more detailed field and modeling studies are needed to develop further refined higher order parameterizations of CHW, we use the simple first-order CHW parameterization of Phillips et al. [2010] in this paper. We evaluate sensitivity to the CHW parameterization by presenting simulations in which $R$ values are varied over a wide range. The end-member cases in these sensitivity studies permit an assessment of the sensitivity of modeled ice temperatures and velocities to a wide range of variation in the strength of CHW.

[10] We present ice temperature and velocity simulations for the Sermeq Avannarleq flow line using a diagnostic thermomechanical model. Our primary objective is to compare simulations with and without CHW, to evaluate the thermomechanical influence of CHW as a possible mechanism for the recent acceleration of inland ice. Our model solves for temperature and velocity fields on a two-dimensional vertical cross-section model along a flow line, based on the shallow-ice approximation. The energy equation for ice is augmented with the aforementioned parameterization of CHW to represent heat transfer from the CHS. The model incorporates variations in the temperature-dependent flow law parameter ($A$) over the model domain. The depth variation of $A$ is particularly important for capturing the influence of significant ice temperature variations with depth, as well as the presence of relatively soft pre-Holocene Wisconsin ice, which is present at depths >680 m in West Greenland [Paterson, 1991; Huybrechts, 1994]. By using measured ice geometry, we avoid any uncertainties stemming from the calculation of ice thickness using a mass balance or continuity equation.

[11] We acknowledge the use of the shallow-ice approximation and a steady state energy equation as limitations of our modeling approach. The limitations of the shallow-ice approximation for modeling ice flow in regions of complex geometry have been discussed previously [Blatter, 1995; Blatter et al., 1998; Dukowicz et al., 2011; Leng et al., 2012; Seddik et al., 2012]. On the Sermeq Avannarleq flow line, the shallow-ice approximation is likely to be most inaccurate near the terminus, where the ice is thin and surface slopes are large. The limitation of a steady state thermodynamic
assumption is discussed further in sections 3 and 5.1 below. Notwithstanding the above limitations, the comparison between thermomechanical models with and without active CHW helps to elucidate the potential response in ice temperature and velocity due to CHW driven by an ascending ELA.

[15] This paper is organized as follows: section 2 describes the field site and observational data sets used in our thermomechanical modeling. Section 3 describes the thermomechanical modeling approach including the governing equations and details of the computational approach. In section 4, we present modeled temperature and velocity fields for Sermeq Avannarleq under a range of assumptions and comparisons to available temperature and velocity data. In section 5, we discuss the potential importance of CHW as a mechanism for the thermomechanical response of an ice sheet and consider alternative mechanisms for the observed velocity increase in the contemporary wet snow zone in Southwest Greenland. We present some concluding remarks in section 6.

2. Field Site and Observational Data Sets

[11] Sermeq Avannarleq is the first tidewater glacier north of Jakobshavn Isbrae in West Greenland, located at 69°25' N and at approximately 49°55' W (Figure 1). Sermeq Avannarleq has been the focus of a number of previous studies conducted by the Geological Survey of Denmark and Greenland and the University of Colorado, including englacial temperature profiles at five locations along the central flow line [Thomsen and Thorning, 1992]. The trajectory of the central flow line in the ablation zone was derived for our present study using interferometric synthetic aperture radar (InSAR) ice surface velocities from 2001 [Joughin et al., 2010a]. In the accumulation zone, flow line trajectory was derived from surface slope data based on a 2008 global Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) digital elevation model made available by United States Geological Survey (USGS; http://www.terrainmap.com/rm39.html).

[14] We use two historical ice temperature profiles (locations TD3 and TD5; Figure 1) as validation data sets for our thermomechanical model. Twenty-five years of continuous climate data are available along the flow line from six automatic weather stations that are part of the Greenland Climate Network. The locations of four of these weather stations are shown in Figure 1 [Steffen and Box, 2001]. Below we describe the InSAR velocity data along the flow line that serves as our primary modeling target. We also describe various input data sets for the thermomechanical model, including ice surface elevation, bedrock elevation, surface mass balance and ELA, surface air temperature, and crevasse spacing derived from visible imagery.

2.1. InSAR Velocity Data

[15] InSAR maps of Greenland Ice Sheet surface velocity have been made available as part of the NASA MEaSUREs project (Making Earth System Data Records for Use in Research Environments) [Joughin et al., 2010a; Joughin et al., 2010b]. RADARSAT-1-derived velocity maps of the ice sheet are presently available for four wintertime epochs: 2000/01 (3 September 2000 to 24 January 2001), 2005/06 (13 December 2005 to 20 April 2006), 2006/07 (18 December 2006 to 15 April 2007), and 2007/08 (1 January 2007 to 31 December 2008). Figure 2 shows the ice surface velocity along the Sermeq Avannarleq flow line for these epochs. Differentiating the 2000/01 and 2007/08 velocity data along the Sermeq Avannarleq flow line indicates a velocity increase of ~40 m a⁻¹ around 85 km upstream from the terminus, where the ice surface elevation is ~1500 m (Figure 3). Assuming an absolute uncertainty of ±10 m a⁻¹ for each InSAR-derived velocity field, this change in velocity exceeds the absolute uncertainty associated with differenting two InSAR-derived velocity fields (i.e., ±20 m a⁻¹) [Joughin et al., 2010a].

[16] The maximum inland extent of the Sermeq Avannarleq annual ice velocity cycle is most likely 70 km upstream from the terminus [Colgan et al., 2012]. Thus, it is unlikely that the apparent differences between 2000/01 and 2007/08 ice velocities in the vicinity of km 85 result from seasonal velocity differences during the satellite sampling periods. The increase in inland ice velocity appears to be distinct from a significant increase in terminus velocity, as a 500 m elevation band of negligible velocity change over the epoch separates the terminus and inland velocity increases (Figure 3). Given the paucity of pre-2005 satellite-derived ice velocities, it is difficult to constrain the precise history of velocity change over the 2000 to 2008 interval. The 95% confidence interval of the rate of change in ice velocity over the 2000/01 to 2007/08 period, assuming an uncertainty of ±10 m a⁻¹ and using an n − 1 approach with n = 4 years of data, suggests that the increase in ice velocity between km

Figure 2. InSAR-derived wintertime surface velocities along the flow line in the ablation zone of Sermeq Avannarleq for 2000/01 (blue), 2005/06 (light blue), 2006/07 (light pink), and 2007/08 (red). The locations corresponding to the 2001 (1400 m) and 2007 (1600 m) ELA are also shown.

Figure 3. The observed change in ice surface velocity between wintertime 2001/02 and 2007/08 versus elevation at Sermeq Avannarleq. One region of velocity increase is apparent at the terminus, while a second region of velocity increase is apparent at 1500 m elevation between km 82 and km 98 in the contemporary wet snow zone.
Greenland, revealed by differencing satellite-derived velocity throughout the contemporary wet snow zone in Southwest Greenland. Revealed by differencing satellite-derived velocity throughout the contemporary wet snow zone in Southwest Greenland. InSAR-derived velocity with the 2000/10 mean ELA, and, in several places, exceeds the absolute uncertainty associated with differencing two InSAR swaths, is closely aligned with the portion of the ice sheet that has recently begun to experience significant surface melt following the transition from historical percolation zone to contemporary wet snow zone (Figure 5). While we acknowledge that the absolute magnitude of this increased ice velocity anomaly is comparable to spatial variations of the difference field, we note that the anomaly spans multiple InSAR swaths, is closely aligned with the 2000/10 mean ELA, and, in several places, exceeds the absolute uncertainty associated with differencing two InSAR-derived velocity fields (i.e., ±20 m a⁻¹) [Joughin et al., 2010a]. Thus, we interpret the increased ice velocity throughout the contemporary wet snow zone in Southwest Greenland, revealed by differencing satellite-derived velocity fields, to represent a meaningful geophysical signal distinct from noise. Previous studies investigating changes in InSAR-derived ice velocity have focused on coastal outlet glaciers, rather than inland ice in the vicinity of the ELA [Joughin et al., 2010a]. For this reason, velocity trends in these inland regions may not have been noticed previously.

### 2.2. Ice Geometry

Our thermomechanical model computes ice temperature and velocity fields for a specified ice geometry at a snapshot in time. Ice surface topography was obtained from a 2008 ASTER digital elevation model made available by USGS for 2007 simulations. The 2001 topography was derived by correcting the 2008 topography with the mean annual ice thickness change observed by Zwally et al. [2005]. Bedrock topography was obtained from National Snow and Ice Data Center (http://nsidc.org/data/0092.html) and the Center for Remote Sensing of Ice Sheets (http://www.cresis.ku.edu/data/greenland). The bedrock topography, available at a nominal resolution of 5 km, was interpolated to 500 m model resolution using a kriging algorithm. There are potential inaccuracies in the bedrock topography due to uncertainties in calculating ice thickness from ice-penetrating radar data. It is plausible that roughness in bedrock topography at scales <5 km is not accurately represented in our estimates of bedrock elevation.

### 2.3. Surface Mass Balance and Air Temperature

The surface ice temperature at each column along the Sermeq Avannarleq flow line is prescribed as the 7 year mean annual air temperature interpolated from National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) Reanalysis output [Kalnay et al., 1996]. The 7 year mean annual air temperature at the flow divide (3220 m) is 239.0 (1990), 239.8 (2001), and 241.1 K (2007), while the 7 year mean annual air temperature at the Sermeq Avannarleq terminus is 272.0 (1990), 273.1 (2001), and 273.1 K (2007).

We interpolate Regional Atmospheric Climate Model (RACMO2) modeled surface mass balance, with nominal 11 km horizontal resolution, along the Sermeq Avannarleq flow line [Ettema et al., 2009]. As complex topography introduces significant discrepancies between RACMO2 surface mass balance and in situ observations near the terminus of Sermeq Avannarleq, we rely on in situ observations downstream of km 25. The RACMO2 output suggests that the highest accumulation rate (60 cm WE a⁻¹) occurs at approximately 1800 m elevation in all three epochs (1990, 2001, and 2007). At the terminus (km 0), we assess negative surface mass balances of 300 cm WE a⁻¹ (1990), 450 cm WE a⁻¹ (2001), and...
just downstream of the ELA. The density of moulins in the suggests that the present crevasse spacing is 44% of the Sermeq Avannarleq ablation zone. Satellite imagery zone between 1985 and 2009. Crevasse surface increased by 13% in the Sermeq Avannarleq ablation 

Figure 6. Surface mass balance along the Sermeq Avannarleq flow during the 1991 to 2001 and 2001 to 2007 epochs. Vertical dashed lines denote equilibrium line altitude (ELA) position in each epoch.

550 cm WE a⁻¹ (2007) based on in situ observations. The ELA was located at ~1470 m elevation (km 82) in 2001 and ~1610 m elevation (km 98) in 2007 (Figure 6).

2.4. Inference of R Values

[21] In this subsection, we describe the rationale behind the approaches used for specifying R values in the wet snow and accumulation zones. We assume that CHW is active in the region from the terminus to an elevation 150 m above the ELA in the wet snow zone (km 100 in 2001 and km 120 in 2007). This is based on the observation that the wet snow zone consists of water-saturated snow and hence experiences both runoff and refreezing [Pfeffer and Humphrey, 1998]. As described by Fountain and Walder [1998], meltwater drains through the snowpack and into crevasses during the melt season. We conceptualize the wet snow zone to be intersected by crevasses, in which meltwater is retained at the end of the melt season. Thus, in the wet snow zone upstream of the ELA, we restrict the influence of CHW to the top 80 m of the ice sheet, which is a reasonable value for crevasse depth [Harper et al., 2010; Clason et al., 2012]. As crevasses extend to a greater depth than the firn, we expect the warming influence of crevasses to exceed that of the firn. Previous models have alternatively incorporated the influence of the latent heat released by refreezing within the firn as a surface source of heat [Baolin et al., 1986]. We have effectively modified this approach to account for water in crevasses up to 80 m deep.

[22] High-resolution satellite imagery offers a potential basis for quantifying crevasse spacing and moulin density at the ice surface in the ablation zone [Colgan et al., 2011b; Phillips et al., 2011]. However, direct in situ measurements of crevasse spacing and moulin distributions below the ice surface in the ablation zone are scarce. Colgan et al. [2011b] observed that the area covered by crevasses >2 m wide at the surface increased by 13% in the Sermeq Avannarleq ablation zone between 1985 and 2009. Crevasses fields presently cover 44% of the Sermeq Avannarleq ablation zone. Satellite imagery suggests that the present crevasse spacing is <100 m on average just downstream of the ELA. The density of moulins in the Sermeq Avannarleq ablation zone is irregular, with a mean of 5.5 km⁻² and a maximum of 12 km⁻² derived from satellite imagery [Phillips et al., 2011]. The surface of the Sermeq Avannarleq ablation zone can therefore be characterized as densely covered by crevasses and/or moulins. Based on these observations, we believe that a strong case can be made for the prevalence of an extensive CHS in the ablation zone, which has also been inferred by in situ radar investigations [Catania and Neumann, 2010].

[23] Surface imagery alone is not sufficient to constrain the geometry of the CHS and provide a specification of R values. While the internal geometry of the CHS at depth is no doubt related to the surface expression of crevasses and moulins, it could be much more complex, exhibiting multidimensional connectivity. The number of clear surface images per season is limited, and substantial portions of the wet snow and percolation zones remained snow covered throughout the year. Another important factor in the parameterization of R values is the availability of meltwater to penetrate the ice and fill crevasses. If meltwater generation is limited, crevasses may not all fill with water. When plenty of meltwater is available, crevasses may penetrate to the bed and not retain water. It is therefore impossible to use satellite imagery as the only basis for parameterizing R values. For these reasons, we took the approach described below. Our approach uses different values of R near the surface (<80 m depth) and in the deeper portions of the ice. For assigning R values at shallow depths, we first estimated R values in the vicinity of TD5 (1140 m elevation at km 49) and TD3 (615 m elevation at km 21), where the surface imagery is clear, and partially confirmed by our in situ observations. Near TD5, the crevasse spacing is about 100 m. However, remotely sensed and in situ observations also suggested that not all crevasses were filled with water. We therefore assigned a value of R = 200 m (i.e., every other crevasse filled with water) at TD5. In the vicinity of TD3, the ice surface is very densely crevassed (also see Figure 1), and we estimated R = 20 m. We then parameterized R values based on local surface slope and elevation between TD5 and TD3. We use surface slope as a controlling variable based on the notion that tensile stresses, which influences crevasse formation, are controlled by local slope. We include elevation in our parameterization as a surrogate for meltwater availability, as surface mass balance is dependent on surface air temperature, which is in turn dependent on elevation. Our parameterization for R is as follows:

\[ R = f(\max(z_s) - z_t)z_s \]  

(1)

[24] where \( z_s \) denotes the ice sheet surface elevation, \( z_t \) is the local surface slope, and \( f \) is a dimensionless scaling factor of 0.25 used to match R values at TD3 and TD5. We prescribe \( R = 200 \) m as the maximum R value upstream of TD5, since the limited imagery available in this region indicated crevasse spacing of the order of 100 m or less in crevassed regions. Downstream of TD3, we maintained a value of \( R = 20 \) m down to the terminus.

[25] To assign R values at depths below 80 m, we used the following scheme for a base case. Between the ELA and 150 m above the ELA, we do not prescribe any crevasses penetrating to the bed. Downstream of the ELA, we assigned every third crevasse to penetrate to the bed along the characteristic englacial hydraulic pressure head slope [Shreve, 1972]. We prescribe this setup to honor previous observations and inferences that only a fraction of crevasses are water-filled and penetrate to the bed [Clason et al., 2012].
Figure 7. The distribution of prescribed $R$ values (average half-spacing between CHS elements) for the base case (s1) in the Sermeq Avannarleq ablation zone (in meters). Different schemes are used for depths <80 m and >80 m, as described in section 2.4. At depths >80 m, the orientation of lines of constant $R$ is along the characteristic englacial hydraulic pressure head slope [Shreve, 1972]. In sensitivity runs, the distribution of $R$ values was significantly perturbed about this base case.

The variation of $R$ values along the flow line in vertical cross section is shown in Figure 7. We acknowledge that there are uncertainties associated with the above approach for assigning $R$ values. For this reason, in the thermomechanical model simulations, we carried out a sensitivity analysis in which $R$ values vary over a wide range of perturbations about the base case: (run s1) “base case” modified to exclude dependence on the local ice surface slope in equation (1) (i.e., linear decrease in $R$ from the TD5 to TD3 location); (run s2) base case with all crevasses penetrating to the bed; (run s3) base case with every second crevasse penetrating to the bed; (run s4) base case with every fifth crevasse penetrating to the bed; (run s5) base case with no crevasses penetrating to the bed (referred to as “surface CHW” in sections 4 and 5); and (run s6) $R$ values maintained constant vertically rather than along the characteristic englacial hydraulic pressure head slope [Shreve, 1972].

3. Thermomechanical Model

[26] Our coupled thermomechanical model is similar to that of Funk et al. [1994], modified to include CHW using the dual-column parameterization described by Phillips et al. [2010]. The model employs the shallow-ice approximation to the momentum equation in combination with mass balance and energy equations as described below. The model calculates the horizontal and vertical velocity and temperature fields on a vertical cross section along a flow line, for specified ice geometry (i.e., a snapshot in time). Strictly speaking, even for a snapshot in time, coupled thermomechanical calculation of velocities and temperatures requires transient solutions of the mass balance and energy equations. The momentum equation does not explicitly involve a time derivative because acceleration terms are neglected. However, velocity fields calculated using the momentum equation are based on evolving geometry and temperature-dependent viscosity in each time step of a transient computation and thus evolve in time. When the ice geometry is specified based on measurements, solution of a transient mass balance equation is not required for a snapshot computation. However, the vertical velocity solution should consistently incorporate both surface mass balance boundary conditions and estimated rates of change in ice surface elevation.

[27] In the absence of CHW, the thermal response time scale to warming air temperatures is centuries to millennia [Johannesson et al., 1989]. Assuming that the ice sheet was in a steady state before the recent climate warming initiated around 1990 in Greenland, a significant thermal response to warming would not be expected. In the absence of CHW, it is thus justifiable to use a one-time coupled solution of the momentum equation, a mass balance equation (with consistently specified surface mass balance and rate of change in ice surface elevation), and a steady state energy equation for calculating horizontal and vertical velocity and temperature fields at a snapshot in time, as in previous studies [Funk et al., 1994]. In the computations presented in this paper, we use a similar approach even in the presence of CHW, mainly to expedite computational efficiency. While the one-time computation of the momentum and mass balance equations is readily justified for a snapshot in time, the use of a steady state energy equation may be inaccurate, because thermomechanical transients induced by CHW persist over time scales of the order of one to three decades [Phillips et al. 2010]. We fully acknowledge that the steady state assumption is a limitation in the computations for the cases with CHW. Although we cannot overcome this limitation completely, we consider a wide range of variation in the strength of CHW in sensitivity studies (see sections 2.4 and 4). In cases where the strength of CHW is reduced, the ice temperatures calculated by the model are colder than in the base case. Colder ice temperatures are also expected during the transient CHW regime, before thermal equilibrium with CHW is attained. Thus, the variation in ice surface velocities among the various sensitivity tests indirectly permits an assessment of thermomechanical feedback during the transient CHW phase.

[28] In this section, we describe the formulation of our thermomechanical model and approaches used for computation. In the following, $x$ denotes a horizontal coordinate,
z a vertical coordinate (positive upward), and \( t \) time. Model computations were implemented using a rescaled vertical coordinate:
\[
\zeta = \frac{z - z_b(x)}{z_b(x,t) - z_b(x)}
\]  
(2)

where \( z_b(x,t) \) and \( z_b(x) \) respectively denote the ice surface and bedrock elevations. The model uses observed values of \( z_b(x,t) \) and \( z_b(x) \) for a snapshot in time \( t \). The ice thickness is denoted as \( H(x,t) = z_b(x,t) - z_b(x) \). The momentum, mass, and energy balance equations were transformed to the \((x,\zeta)\) coordinate system and then discretized. The momentum equation based on the shallow-ice approximation is as follows:
\[
\frac{\partial w}{\partial \zeta} = 2A(\zeta, \theta) \left( \rho_i g \sin \alpha \right)^n H^{n+1} (1 - \zeta)^n
\]  
(3)

where \( u \) is the horizontal ice velocity, \( \rho_i \) is the density of ice, \( g \) is the acceleration due to gravity, \( \alpha \) is the ice surface slope, and \( n = 3 \) is the flow law exponent [Glen, 1958]. The ice surface slope \( \alpha \) in equation (3) is calculated over one ice thickness, as recommended when the shallow-ice approximation is used [Hutter, 1983]. The ice surface slopes over one ice thickness employed in the thermomechanical model are therefore distinct from the ice surface slopes used to parameterize \( R \), which are determined over a shorter horizontal distance than one ice thickness. The flow law parameter for ice \( A(\zeta, \theta) \) is represented as a function of elevation and ice temperature \( \theta_i \) which varies both horizontally and vertically along the flow line:
\[
A(\zeta, \theta) = E(\zeta) A_0 \exp \left( - \frac{Q_0}{R_G \theta_i} \right)
\]  
(4)

In equation (4), \( E(\zeta) \) is an enhancement factor, \( A_0 \) is a reference value of \( A \), \( Q_0 \) is the creep activation energy of ice, and \( R_G \) is the universal gas constant. Unlike Funk et al. [1994], we do not fit the enhancement factor to match observed ice surface velocities. We use \( E = 3 \) at depths \((z - z) = H(1 - \zeta)\) 680 m to account for the presence of softer Wisconsin basal ice observed in the Greenland ice sheet [Lüthi et al., 2002]. Closer to the terminus of the flow line \((< 38.5 \text{ km})\), where \( H < 680 \text{ m} \), no Wisconsin basal ice is present, and \( E = 1 \). The temperature dependence of \( A \) in equation (4) is based on the approach of Marshall et al. [2005], with parameter values \( A_0 = 1.14 \times 10^5 \text{ Pa}^{-3} \text{ a}^{-1}, Q_0 = 60000 \text{ J mol}^{-1} \) for \( \theta_i \leq 263.14 \text{ K} \), and \( A_0 = 6.47 \times 10^{10} \text{ Pa}^{-3} \text{ a}^{-1}, Q_0 = 139000 \text{ J mol}^{-1} \) for \( \theta_i > 263.14 \text{ K} \).

The mass balance equation, or incompressibility condition, written in the \((x,\zeta)\) coordinate system is as follows:
\[
\frac{\partial w}{\partial x} + \frac{1}{H} \left( \frac{\partial \zeta}{\partial \zeta} \frac{\partial w}{\partial \zeta} \right) + \frac{1}{H} \frac{\partial w}{\partial \zeta} = 0
\]  
(5)

where \( w \) denotes the vertical ice velocity, which can be obtained by integrating equation (5) and using the kinematic condition at the ice surface:
\[
w_z = -b + u_t \frac{\partial z}{\partial x} + \frac{\partial s}{\partial t}
\]  
(6)

In equation (6), \( u_t \) and \( w_z \) respectively denote the horizontal and vertical components of the ice surface velocity, and \( b \) denotes the accumulation/ablation rate at the surface (positive for accumulation). The resulting integrated expression for \( w \) as follows:
\[
w = -b + \frac{\partial z}{\partial t} + u \left( \frac{\partial \zeta}{\partial \zeta} + \frac{\partial H}{\partial \zeta} \right) + \int_0^\zeta \frac{\partial}{\partial \zeta} (Hu) \, d\zeta
\]  
(7)

The steady state energy equation written in the \((x,\zeta)\) coordinate system is as follows:
\[
u \frac{\partial \theta_i}{\partial x} + \frac{\theta_i}{H} \frac{\partial H}{\partial \zeta} \left( \frac{\partial \zeta}{\partial \zeta} + \frac{\partial H}{\partial \zeta} \right) + \frac{\partial (\zeta + \frac{\partial H}{\partial \zeta})}{\partial \zeta} \frac{\partial \theta_i}{\partial \zeta} - \kappa \frac{\partial^2 \theta_i}{\partial \zeta^2} = \frac{Q}{\rho_i c_i} + \kappa \frac{\partial (\theta_{PMP} - \theta_i)}{\partial \zeta}
\]  
(8)

In equation (8), horizontal heat conduction is neglected, as is common in ice sheet thermodynamic models, because horizontal energy transport is dominated by advection (first term in (8)) [Funk et al. 1994]; \( \kappa \) and \( c_i \) respectively denote the thermal diffusivity and heat capacity of ice; \( Q \) denotes the strain-heating rate; \( \theta_{PMP} \) denotes the pressure melting point temperature, and \( R \) is a characteristic half-spacing between elements of the cryo-hydrologic system. Each of these terms is described further below. The vertical advection term in equation (8) involves a rescaled vertical velocity (defined as \( w \) in equation (9) below), which can be evaluated using equation (7) as follows:
\[
w' = \left( \frac{w - \zeta \frac{\partial H}{\partial \zeta}}{H (1 - \zeta)} + \frac{u}{H} \left( \frac{\partial \zeta}{\partial \zeta} + \frac{\partial H}{\partial \zeta} \right) \right) 1 + \frac{1}{H} \frac{\partial (Hu)}{\partial \zeta}
\]  
(9)

It is readily shown that on the surface, \( w'(\zeta = 1) = -b/H \). Integrating the mass balance equation (5) from the bed to the surface, it is also readily shown that on the bed, \( w'(\zeta = 0) = -b_m/H \), where \( b_m \) denotes the basal melt rate (which is zero in regions where the bed is cold and there is no basal melt). In temperate bed regions, the basal melt rate is computed by assuming that the geothermal heat flux is entirely used to produce basal melt [Funk et al., 1994]. Initial tests showed that the horizontal velocity and temperature fields calculated using equation (9) versus a linear approximation to \( w' \) between the values of \(-b_m/H \) and \(-b/H \) at \( \zeta = 0 \) and \( 1 \) were almost identical. For this reason, we employed a linear approximation to \( w' \) in the computations, which speeds up the thermomechanical iteration significantly.

The strain-heating rate is represented following the standard approach employed with the shallow-ice approximation as follows:
\[
Q(\zeta) = 2A(\zeta, \theta) \left( \rho_i g \sin \alpha \right)^n H^{n+1} (1 - \zeta)^n
\]  
(10)

The last term in equation (8) represents energy input due to CHW based on the parameterization of Phillips et al. [2010], with the CHS assumed to continuously be at the pressure melting point \( \theta_{PMP} \) in the steady state energy equation.
This assumption is supported partially by observations that indicate persistence of liquid water throughout the winter in the Sermeq Avannarleq ablation zone [Catania and Neumann, 2010]. Additionally, the fully transient model calculations of Phillips et al. [2010] suggest that after several cycles of seasonal meltwater input, some fraction of liquid water remains in the CHS throughout subsequent winters. This provides theoretical support for the notion that the CHS temperature is at the pressure melting point year round. However, as noted above, the use of a steady state energy equation for simulations with CHW is a limitation of our modeling approach.

[39] Calculation of velocities and temperatures along the flow line proceeds downstream from the divide, one column at a time. In each column of the thermomechanical model, equations (3), (4), and (8) and a linear approximation to equation (9) are iteratively used to solve for mutually consistent velocity and temperature fields. The ice thickness is discretized into 251 nodes between \( \zeta = 0 \) and 1 at each column. First, an initial guess for \( \theta_i \) is used to determine \( A(\zeta, \theta_i) \) in equation (4), and equation (3) is used to calculate the horizontal velocity profile \( u \) by trapezoidal integration starting from the bed \( \zeta = 0 \), where the boundary condition is \( u = 0 \) if the bed is cold or \( u = u_b \) if the bed is temperate. Typically, a long continuous stretch of temperate bed is predicted upstream from the terminus, and in some simulations additional discontinuous patches of temperate bed occur farther upstream. Basal sliding is prescribed only in the continuous stretch of temperate bed upstream of the terminus. In this stretch, the sliding speed \( u_b \) is prescribed as a linear increase from 0 to 15 m a\(^{-1}\) over the first 10 km and maintained at 15 m a\(^{-1}\) downstream to the terminus thereafter, consistent with independent estimates of the magnitude of basal sliding at Sermeq Avannarleq [Colgan et al., 2012]. Subsequently, the linear profile of \( w^* \) discussed above is used together with \( u \) to obtain a new estimate of \( \theta_i \) by solving a steady state form of equation (8).

[40] The horizontal advection term in equation (8) is approximated using an upstream difference approximation, thus involving \( \theta_i \) from the upstream column (which has already been calculated; for the first column at the divide, \( u = 0 \) over the entire depth, and the horizontal advection term is dropped). A finite-difference discretization of equation (8) is used, which produces a tri-diagonal system of equations for \( \theta_i \) at the computational nodes. Boundary conditions for \( \theta_i \) correspond to a specified mean air temperature at the ice surface (\( \zeta = 1 \)) and a constant geothermal heat flux of 47 mW m\(^{-2}\) [Fahnestock et al., 2001] at the bed (\( \zeta = 0 \)) for cold bed conditions. The treatment of temperate bed conditions is discussed further below. In subsequent iterations, the updated estimates \( \theta_i \) are used to recalculate \( u \), and the sequential iteration is continued until there are insignificant changes in \( u \) and \( \theta_i \). At this stage, the computation has converged to a mutually consistent set of velocity and temperature fields on the column, and we proceed to the next column.

[41] If the \( \theta_i \) solution in any iteration produces temperatures above \( \theta_{PMP} \), the solution is revised by invoking the presence of a cold-temperate transition surface (CTS) following the approach of Funk et al. [1994]. For the conditions representative of Sermeq Avannarleq, temperate conditions typically occur in the lower regions of the ice column close to the bed, similar to the simulation results of Funk et al. [1994] for Jakobshavn Isbrae. In the accumulation zone, where vertical advection is downward, \( \theta_i \) values \( > \theta_{PMP} \) are set equal to \( \theta_{PMP} \), and the excess energy is assumed to be used towards producing a liquid water content fraction within the temperate ice. In the ablation zone, where vertical advection is upward, the location of the CTS (\( \zeta_{CTS} \)) needs to be determined iteratively (i.e., using a nested iteration during the temperature solution, which is embedded within the overall thermomechanical iteration) by satisfying two conditions simultaneously: the temperature at the CTS should equal the pressure melting point, and continuity of upward energy flux must be maintained across the CTS. In each nested iteration, \( \theta_i \) at nodes below the current estimate of the CTS location (including the node at the bed) are set to \( \theta_{PMP} \), and the solution for \( \theta_i \) at nodes at and above the CTS is obtained by using the condition of continuity in upward energy flux at the CTS as an internal boundary condition. An additional moving node is introduced in the computation, corresponding to \( \zeta_{CTS} \). The energy flux continuity condition can be written in the \( \zeta \) coordinate using the rescaled velocity \( w^* \) in the following form [Funk et al., 1994]:

\[
- \frac{k}{H^2} \frac{\partial \theta_i}{\partial \zeta} \bigg|_{CTS} + \frac{k}{H^2} \frac{\partial \theta_{PMP}}{\partial \zeta} + \rho_w w^* \zeta \Delta L = 0
\]

[42] In equation (10), the subscript “CTS” denotes values evaluated at \( \zeta_{CTS} \), the superscript “m” denotes the upper or cold side of the CTS; \( k \), \( \rho_w \), \( L \), and \( \mu \) respectively denote the thermal conductivity of ice, density of water, latent heat of fusion, and the volumetric water content fraction in temperate ice (a value of \( \mu = 0.01 \) is assumed [Duval and LeGac, 1977]). The left side of equation (10) is the conductive heat flux into cold ice, and the right side represents the energy released by freezing of the liquid water content in temperate ice less the small amount of energy conducted into temperate ice from the CTS, which results because \( \theta_{PMP} \) decreases with depth. The nested CTS iteration converges when the calculated temperature at the bottom of the cold (i.e., upper) portion of the ice column equals the pressure-melting-point temperature at that vertical location, i.e.,

\[
\theta_i^+ (\zeta_{CTS}) = \theta_{PMP} (\zeta_{CTS})
\]

[43] At this point, the correct location (\( \zeta_{CTS} \)) has been determined, simultaneously honoring both conditions in equations (10) and (11). It should be noted that when temperate conditions are encountered, the nested iteration to locate \( \zeta_{CTS} \) is repeated during each temperature solution within the overall thermomechanical iteration.

4. Results

[44] Our validation observations for the thermomechanical model are the 1990 ice temperatures at boreholes TD3 and TD5, while the 2001 and 2007 ice surface velocities provide our ultimate modeling target. Comparing the extent of temperate bed conditions predicted by the model with contemporary observations of the upstream distance from the terminus to which seasonal acceleration extends (inferring the presence of an active subglacial hydrology system and thus temperate bed conditions) also serves as an indirect validation.
Figure 8. Simulated 1990 ice temperature profiles compared to the observed 1990 ice temperature profiles at (a) TD3 and (b) TD5. The temperatures predicted by the no CHW model (blue) underestimate the observed temperatures (black symbols), while the base case simulations with CHW (red) match the observed ice temperature profiles more closely.

For the 2001 and 2007 simulations, we show detailed results for three main scenarios: (i) “no CHW”, (ii) the “base case CHW” scenario described in section 2.4, and (iii) an extreme end-member surface CHW in which no crevasses penetrate to the bed. We also show simulated surface velocities for a range of other scenarios (described in section 2.4) to demonstrate the sensitivity to varying strengths of CHW.

The only presently published vertical ice temperature profiles at Sermeq Avannarleq were obtained in 1990 [Thomsen and Thorning, 1992]. Although velocity measurements are not available for 1990, it is still useful to compare ice temperatures predicted by the thermomechanical model with observations. For this reason, we simulated conditions representative of 1990 in an effort to validate our modeled ice temperatures. Ice temperature calculations based on the thermomechanical model for 1990 at boreholes TD3 and TD5 are shown in Figure 8. TD3 was located well within the ablation zone in 1990, while TD5 was at the historical ELA (Figure 1). For the 1990 simulations, the base case CHW was prescribed from 100 m above the 1990 ELA down to the terminus, using $R$ values derived from equation (1), according to the base case described in section 2.4. Like much of the Greenland Ice Sheet, Sermeq Avannarleq was in approximate geometric equilibrium in 1990 [Rignot et al., 2008]. Because much of the rapid warming of air temperatures and upward ascent of the ELA happened after 1990, we also expect that Sermeq Avannarleq was in thermodynamic equilibrium at that time, except at near-surface depths that are influenced by the annual air temperature cycle (~10 m). For this reason, the 1990 ice temperatures serve as a robust validation target for our thermomechanical model, because the use of a steady state energy equation is justifiable. Ice temperature measurements in the nearby Jakobshavn glacier from 1990 were also interpreted using a thermomechanical calculation that employed a steady state thermodynamic model [Funk et al., 1994; Lüthi et al., 2002].

At TD3, simulations without CHW do not capture the general shape of the observed ice temperature profile, while simulations with CHW closely reproduce the observed ice temperature profile (Figure 8a). A more detailed parameterization of the depth dependence of $R$ could potentially further improve the agreement between modeled and observed ice temperatures (i.e., warmer conditions at 50 to 150 m depth). The simulation with base case CHW also reasonably replicates the measured temperature profile at TD5, including a local warming at 80 m depth that likely reflects the influence of water-filled shallow crevasses [Jarvis and Clarke, 1974]. In contrast, the no CHW simulation produces much colder profiles than observed, with minimum ice temperatures nearly 10°C colder than observed (Figure 8b). The 1990 ice temperature simulation thus strongly supports the notion that CHW was active in the ablation zone of Sermeq Avannarleq prior to 1990 and influenced the ice temperature distribution along the flow line under geometric and thermodynamic equilibrium conditions, prior to the onset of recent climate change.

By 2001, Sermeq Avannarleq was no longer in dynamic or geometric equilibrium; the terminus was retreating by an average of 20 m a$^{-1}$ [Colgan et al., 2011b], and the surface mass balance was negative [Ettema et al., 2009]. In Figures 9a through 9c, the ice temperature and velocity profiles for three simulations for 2001 are shown: (i) no CHW, (ii) surface CHW—i.e., only shallow water-filled surface crevasses (same as sensitivity run s5 below), and (iii) base case CHW. In Figure 9a, the modeled ice temperature distribution infers cold ice far into the ablation zone. Temperate conditions at the bed and hence basal sliding are predicted to occur only downstream of km 18. In Figure 9b, the modeled ice temperature distribution exhibits a warm layer close to the ice sheet surface. This is due to the surface CHW parameterization corresponding to 80 m deep water-filled crevasses in the wet snow and ablation zones (downstream of km 82). The average value of the half-spacing $R$ is about 192 m in the wet snow zone. The ice temperatures and velocities deeper than ~100 m and extent of temperate bed conditions in Figure 9b are however nearly identical to the no CHW simulation shown in Figure 9a. Compared to the no CHW simulation, the velocity profile for the surface CHW simulation only indicates a slightly higher ice surface velocity downstream from km 105, near the lower boundary of the wet snow zone. In the base case CHW simulation (Figure 9c), the influence of CHW is most notable, with significantly higher ice temperatures across the full ice sheet thickness and temperate bed conditions inferred as far upstream as km 80 (only 3 km downstream from the 2001 ELA). The region between km 85 and km 45 exhibits increased velocities through the full ice thickness, unlike the no CHW and surface CHW simulations in Figures 9a and 9b. This behavior is consistent with warmer ice temperatures near the bed, which reduces ice viscosity and increases the deformational component of the velocity. The deformational velocity increase is further augmented by basal sliding over the significantly longer stretch of temperate bed.

The 2007 no CHW, surface CHW, and base case CHW simulations are shown in Figures 9d through 9f. Similar to the 2001 simulations, the depth and extent of temperate ice is
Figure 9. Ice temperature (top) and velocity (bottom) profiles on a vertical cross section along the terminal 145 km of the Sermeq Avannarleq flow line for six cases: (a) no CHW in 2001, (b) surface CHW in 2001, (c) base case CHW in 2001, (d) no CHW in 2007, (e) surface CHW in 2007, and (f) base case CHW in 2007. Temperatures are plotted as a difference from the local pressure melting point to highlight regions of temperate bed. Color scales for temperature and velocity variations appear below the figures.

Figure 10. The ice surface velocity profile for each of the six cases shown in Figure 9 overlaid on the wintertime 2001/02 (blue circle symbols) and 2007/08 (green circle symbols) InSAR-derived ice surface velocities.

much greater in the base case CHW simulations than in the no CHW and surface CHW simulations. In contrast to the 2001 simulations, however, the 2007 simulations have a higher ELA (1610 versus 1470 m), and thus the ablation zone has expanded further upstream. The 2007 no CHW temperature distribution (Figure 9d) is not significantly different from the 2001 no CHW case (Figure 9a). The minor differences between Figures 9a and 9d are due to differences in the surface temperature boundary conditions and surface mass balance (which influences the temperature distribution indirectly through the vertical advection velocity in the energy equation). Admittedly, these differences are to some extent an artifact of using a steady state energy equation with different boundary conditions and surface mass balance. If a transient energy equation were used, the differences would be much smaller. It is reassuring that the differences between the temperature distributions in Figures 9a and 9d are rather small, because this implies that rapid changes in ice geometry, surface temperatures, and mass balance cannot produce significant temperature changes over short durations.

In the 2007 simulations, the region with temperate bed conditions is downstream of km 26 for the no CHW and surface CHW simulations and km 95 for the base case CHW simulation. Comparing the base case CHW temperature distributions (Figures 9c and 9f), there is an upward expansion of warmer temperatures produced by CHW, and the upstream limit of inferred temperate bed conditions expands upstream by about 15 km between 2001 and 2007. Comparing Figures 9c and 9f, it is also apparent that the region of increased ice velocities has expanded further upstream in response to warmer ice temperatures resulting from upward expansion of CHW and temperate bed conditions. The temperature differences between Figures 9c and 9f are no doubt influenced by the assumption that thermal equilibrium in response to the
The upward expansion of CHW has been achieved. In the sensitivity runs shown below, the strength of CHW is varied, and the cases of lower strengths than the base case (i.e., increased $R$ values), are associated with lower temperatures. The surface velocity results for these cases provide a partial assessment of the behavior expected when full thermal equilibrium to transient CHW has not yet been achieved.

In Figure 10, the ice surface velocities from the individual 2001 and 2007 simulations are shown together with InSAR-derived surface velocity data for wintertime 2001/02 and 2007/08. We performed a $t$ test to evaluate the similarity of the velocities for 2001/02 and 2007/08 between km 115 and 45 and find a $p$ value of $2 \times 10^{-10}$ at the 99% confidence level. Hence, the null hypothesis that the two data sets have the same velocity distribution is rejected, and we conclude that the differences between 2001/02 and 2007/08 velocity distributions are statistically significant. All six simulations show nearly identical surface velocity profiles in the accumulation zone upstream of km 135 (not shown in Figure 10 but evident from Figure 11 below). This is consistent with the relatively little change in ice geometry and absence of CHW in the accumulation zone. All simulations appear to underestimate the surface velocity peak between ~km 140 and 120, which shows up in the 2001 and 2007 InSAR velocity data. We believe that this is likely due to uncertainties in the bedrock elevation, as discussed further below. The small differences between the 2001 and 2007 no CHW simulations largely stems from changes in ice geometry between 2001 and 2007. As noted above (Figure 9), the temperature differences between these two simulations are relatively small. The differences between the 2001 and 2007 surface velocities predicted by the no CHW simulations are $<10$ m a$^{-1}$, except downstream of km 30 near the terminus. The surface CHW simulations for 2001 and 2007 produce surface velocities that are only slightly higher than the corresponding no CHW simulations ($<4$ m a$^{-1}$ except for at the terminus). Both the no CHW and the surface CHW simulations significantly underestimate velocities in the wet snow zone and ablation zone, between about km 110 and km 45. Between km 80 and km 45, the surface CHW and no CHW simulations underestimate the observed velocities by about 30 and 60 m a$^{-1}$ in 2001 and 2007, respectively. In both 2001 and 2007, the base case CHW simulations produce ice surface velocities that match observations reasonably well between km 115 and km 45. The upstream expansion of the region of increased ice surface velocities between 2001 and 2007 is also well reproduced by the base case CHW simulations. As noted previously, the upstream limit of CHW was assigned at km ~100 in 2001 and km ~120 in 2007. The deviations of the base case CHW surface velocities from the other two locations begin approximately at these locations in Figure 10. All simulations miss the steep dip in velocity around km 30 near the terminus and exhibit deviations from the InSAR velocities downstream of km 30. In this region, the ice thickness is small, and the surface slope is quite large. Previous studies have suggested that shallow-ice approximation will significantly overestimate ice velocities [Seddik et al., 2012; Dukowicz et al., 2011] under these conditions. For this reason, we do not believe that the poorer agreement between simulated and observed velocities downstream of km 30...
detracts from the reasonable agreement achieved in the base case CHW simulations between km 110 and 45, where the ice thickness is very large (700 to 1200 m) and slopes are much smaller.

[51] As noted in section 2.4, we carried out sensitivity tests by specifying variations in $R$ values about the base case (Figure 7). Although we do not show detailed results (ice velocity and temperature on the cross section of the flow line) for these cases, we show the simulated 2007 ice surface velocities for all these cases in Figure 11. The base case surface velocity that was shown in Figure 10 is indicated as a bold line in Figure 11 (black in the accumulation zone and red in the wet snow and ablation zones). All cases produce identical velocities in the accumulation zone where there is no CHW. The modeled surface velocities range from the surface CHW lower bound (run s5) to the upper bound corresponding to all crevasses penetrating fully to the bed (runs s2). The latter is clearly unrealistic; recent studies suggest that only a fraction of crevasses receive enough meltwater supply to reach the bed [Clason et al., 2012]. Other observations [e.g., Catania and Neumann, 2010] provide support for a CHS that extends to significant depths, a feature not captured by assigning CHW only up to 80 m depth, as in the surface CHW simulations. The cases of every other crevasse and every fifth crevasse penetrating to the bed (as opposed to every third crevasse in the base case) produce a significant spread about the base case. The maximum difference in surface velocity between these two cases is 50 m a$^{-1}$ at km 80.

Assignment of $R$ values at depth by maintaining constant $R$ along the vertical rather than along the characteristic englacial pressure head gradient [Shreve, 1972] does not produce much change in the ice surface velocities. The ice surface velocities are much more sensitive to spacing between englacial passages in the deeper portions of the ice sheet than to their orientation. Ignoring the role of slope in modulating crevasse density in equation (1), i.e., using a linear decrease in $R$ with elevation, does not produce significant differences from the base case. As a final sensitivity test, we considered the influence of errors in estimates of ice thickness or bedrock elevation. Such errors may result from measurement errors in ice thickness measurements and the relatively low resolution (5 km) of the bedrock digital elevation model used in our study [Stoosius and Herzfeld, 2004]. Field observations in boreholes along the flow line in the ablation zone of Sermiaq Avannarleq have suggested significant discrepancies between radar measured ice thicknesses and actual ice thicknesses. To assess the sensitivity of our thermomechanical model to uncertainty in ice thickness, we performed a base case CHW simulation in which ice thickness was increased by 5% (by lowering the bedrock elevation) and found that ice surface velocities increased by 16% (up to 8 m a$^{-1}$) in some portions of the accumulation zone. At km 130, where all the simulations shown in Figure 10 underestimated the surface velocity, the +5% ice thickness simulation produces better agreement with observations. At the same time, the surface velocity for this case and the base case are quite similar in the ablation zone, where the ice thickness is significantly smaller than in the accumulation zone. For this reason, we believe that the uncertainty in bedrock elevation is the most likely cause for the underestimation of surface velocities in the vicinity of km 130.

5. Discussion

5.1. Support for CHW

[55] The 1990 ice velocities and temperatures predicted by our thermomechanical model constitute a valid representation of Sermiaq Avannarleq’s ice flow prior to the recent increase in air temperatures and meltwater production circa 1990 [Overpeck et al., 1997], because the use of a steady state energy equation is justifiable for that epoch. The 1990 ice temperatures predicted by simulations without CHW for two borehole locations, TD3 and TD5 (Figures 8a and 8b), are significantly less than measured temperatures. This suggests a missing source of heat, which we claim to be CHW. The ice temperatures calculated by incorporating CHW agree reasonably well with the measured ice temperatures, providing support for CHW as an important component of the thermodynamics in the ablation zone. As noted above, the use of a steady state energy equation in our computations is not a limitation in the context of predicting temperatures in 1990. In the absence of CHW, the thermal response of ice sheets would be relatively slow, centuries to millennia, and a single steady state temperature calculation would be considered quite accurate for the comparatively brief 1990 to 2010 period. The relatively minor differences in the calculated temperature fields (Figures 9a and 9d) for the no CHW case in 2001 and 2007 are consistent with this expectation. The velocity results obtained for the no CHW case in 2001 and 2007 show differences because of changes in the surface mass balance and ice geometry, but these differences are small. All the results presented above that do not account for CHW therefore represent best estimates of ice velocities and temperatures that can be obtained with conventional shallow-ice thermomechanical models. Thus, the fact that the simulated 2001 and 2007 velocities obtained without CHW greatly underestimate observed velocities strengthens the case for CHW.

[54] Although a transient calculation would better capture the response of ice velocity to CHW between 2001 and 2007, we believe that our steady state calculations provide a reasonable approximation to the present-day temperature field in some portions of the flow line: (i) a steady state representation of the accumulation zone (where CHW is not active) is reasonable because its thermal response is typically slow; and (ii) downstream of the 1990 ELA horizontal location of the Sermiaq Avannarleq ablation zone, where the ablation zone experienced melt inputs long before 1990 and was probably in a steady state influenced by CHW. The steady state energy equation with CHW will overestimate temperatures in the areas near the ELA that have only recently begun to experience melt. As noted above, our sensitivity tests across a wide range of variations in $R$ values encompass a broad range of ice temperatures corresponding to different strengths of CHW. The corresponding range covered by the modeled velocities is shown in Figure 10 and provides a sense of the thermomechanical response to different strengths of CHW, which encompass the lower temperatures that would be associated with transient CHW. We acknowledge here that precise assignment of $R$ values is hampered by the poorly constrained internal geometry of the CHS. In fact, it is hard to say even whether the base case underestimate or overestimates $R$ values; it has been recently suggested that the density of englacial passages may in fact
be much higher than previously expected [Fountain and Creyts, 2012]. Based on Figure 11, we note that all but the two extreme end-members (surface CHW, s5, and the case with all crevasses penetrating to the bed, s2) produce reasonable enhancements of the ice surface velocity compared to the no CHW simulations in the region between km ~ 100 and km ~ 45. Any of these cases may be considered as plausible representations of the consequences of CHW along the flow line over the 1990–2010 period.

[55] For both 2001 and 2007, the base case CHW simulations match InSAR-derived ice velocities despite the large values of the half-spacing $R$ assigned near the ELA ($R = 200$ m near the surface and 600 m below a depth of 80 m). Both InSAR-derived velocity fields (2001/02 and 2007/08) exhibit a strong velocity increase above the ELA. The 2007 surface velocity is about 40 m a$^{-1}$ higher than the 2001 surface velocities between the ELA locations for 2001 and 2007 (km 98 and km 83). The base case CHW simulation and the various sensitivity runs where $R$ values were varied produce varying increases in ice temperatures down to significant depths, thus decreasing its viscosity and allowing significantly greater deformation of the ice closer to the bed. The associated increase in deformational velocity results in an increased surface velocity. Compared to the no CHW and surface CHW cases, the increased warming at higher depths in the base case CHW case produces temperate bed conditions farther upstream (km 82 and km 95 respectively in 2001 and 2007). The higher ice surface velocities in the base case CHW simulation result from both an increase in deformational velocity and the $15$ m a$^{-1}$ prescribed value for the basal sliding velocity. We consider the observation of a seasonal velocity cycle at the University of Colorado/Swiss Federal Institute of Technology (CU/ETH) (“Swiss”) Camp (km 48) [Zwally et al., 2002] as independent evidence for temperate bed conditions at that location. The no CHW simulations produce temperate bed conditions only up to about km 20. This discrepancy also provides support for the importance of CHW.

5.2. Other Possible Mechanisms of Acceleration

[56] It is pertinent to briefly consider other possible mechanisms that may be responsible for the apparent widespread acceleration of the wet snow zone throughout Southwest Greenland. First, we consider the possible consequences of changing ice geometry in the absence of CHW. Between 1985 and 2009, the ice surface slope increased from 1.45 to 1.55 in the vicinity of the ELA at Sermeq Avannarleq. Simultaneously, the ice thickness in this region decreased by 65 m. The net effect of these two counteracting effects is unlikely to produce a significant increase in deformational ice velocity, let alone a 40 m a$^{-1}$ increase as indicated in Figure 3. This is further confirmed by comparing our no CHW simulations for 2001 and 2007 in Figure 10. As noted above, the differences between the velocities calculated for these two epochs stems largely from changes in ice geometry and surface mass balance and is $<10$ m a$^{-1}$ in the region between km 100 and km 45. Next, we consider the possibility that basal sliding alone is responsible for the 40 m a$^{-1}$ observed increase in ice velocity. If we simply add basal sliding to the no CHW simulations to produce ice surface velocities that match the observations, basal sliding must be introduced at km 147. Prescribing basal sliding at this location produces a thermomechanical paradox because the bed is not temperate in this region and cannot support a sustained subglacial hydrologic system. Furthermore, when the enhanced horizontal advection of cold ice due to basal sliding (without a concomitant increase in strain heating) is accounted for, the predicted bed temperatures would be even colder and significantly below the pressure melting point (Figure 12).

[57] A final alternative mechanism that we consider for increased inland ice velocities is the upstream propagation of a loss of terminus back stress by longitudinal coupling [Howat et al., 2008; Joughin et al., 2008; Price et al., 2008]. Previous modeling of the Sermeq Avannarleq flow line suggested that depth-averaged longitudinal coupling stress decreases to less than 10% of the total driving stress upstream of an icefall at km 6, and thus local melt-acceleration.
rather than the inland propagation of a terminus perturbation, was likely responsible for the annual velocity cycle observed at Swiss Camp [Zwally et al., 2002; Colgan et al., 2012]. The relation between the 2001/02 to 2007/08 velocity change versus elevation at Sermeq Avanarleq supports the notion that inland changes in velocity are disconnected from changes in terminus velocity (Figure 3). In the broader context of Southwest Greenland (Figure 5), we also note that the majority of the ice sheet margin in Southwest Greenland is land terminating, and thus a large portion of the wet snow zone that is exhibiting an increase in ice velocity is not susceptible to tidewater terminus perturbation. Following the consideration of these alternative mechanisms for an increase in inland ice velocity, we contend that a significant increase in ice temperature and subsequent decrease in ice viscosity are a highly plausible explanation for the broad increase in velocity observed in the wet snow zone of Southwest Greenland.

6. Concluding Remarks

[58] Our results strongly suggest that CHW is contributing to an increase in ice surface velocity in the vicinity of the ELA in Southwest Greenland. Although the precise quantification of the increase in velocity due to CHW is hampered by the wide range of variation evident from our sensitivity tests where R values are varied, we note that the no CHW simulations underestimate ice velocities significantly (by ~30 to 60 m a\(^{-1}\)). Most simulations that incorporate CHW down to a significant depth produce predictions of ice surface velocities that agree reasonably with observations, with the exception of the highly unrealistic case where all crevasses were assumed to penetrate fully to the bed. The increased ice velocities due to CHW result from a combination of increased deformational velocity and an upstream increase in the extent of temperate bed conditions, which facilitates basal sliding. Unlike other mechanisms that produce largely seasonal accelerations (basal sliding [Zwally et al., 2002, Joughin et al., 2008; Colgan et al., 2012]) or accelerations that extend over a limited distance upstream from the terminus (terminus retreat and elimination of back stress [Joughin et al., 2008]), CHW explains sustained and widespread acceleration that accompanies an ascending equilibrium line. Furthermore, CHW produces a thermomechanically consistent explanation for increased ice flow. Whereas other mechanisms typically overlook enhanced advective cooling and associated negative feedback on deformational velocities that would accompany acceleration, CHW provides an extraneous source of warmth (over and above strain heating) that counteracts advective cooling. Although temperature measurements on Sermeq Avanarleq are presently unavailable, recent observations at Russell Glacier (~250 km south of Sermeq Avanarleq) suggest warmer temperatures than expected from conventional thermodynamic modeling, potentially implicating the influence of CHW [Harper et al., 2010].

[59] As the ELA migrates upward over the Greenland Ice Sheet, CHW will potentially allow both land and marine-terminating glaciers to rapidly achieve new equilibrium thermomechanical states, primarily characterized by elevated ice temperatures and velocities. Our model predicts that inland ice velocities will continue to increase as the ELA migrates upstream due to warming air Arctic temperatures.

Abundant crevasses and moulins are presently available to serve as meltwater entry points to the CHS along a broad stretch of West Greenland [Grenfell, 2000]. We therefore contend that the influence of CHW should be considered, together with other mechanisms, in predicting the future behavior of outlet glaciers in Greenland. For example, Ván der Veen et al. [2011] suggested that a decrease in lateral drag, likely due to CHW in lateral shear zones, may be as, or more, important as a loss of terminus back stress in explaining the recent acceleration of Jakobshavn Isbrae.

[60] We again acknowledge that the use of the shallow-ice approximation and a steady state energy equation are limitations of our modeling approach. The use of a steady state energy equation does not negate our assessment that thermomechanical models without CHW underestimate ice velocities significantly. However, fully transient thermomechanical models incorporating CHW will provide a more rigorous assessment of the thermomechanical consequences of CHW and help to constrain response time scales. The sensitivity tests shown in Figure 9 across a wide range of R values clearly show the influence of varying CHW strengths (which are reflected in varying ice temperatures) on simulated ice surface velocities. It is humbling to acknowledge that the internal geometry of the CHS is rather poorly understood, and at present it is very difficult to conclusively state which of the schemes used for assigning R values among the base case and sensitivity tests come closest to reality. Further investigations to characterize the internal geometry of the CHS will greatly help to improve the parameterization of CHW.

[61] Acknowledgments. We would like to thank the editor at JGR and three anonymous reviewers for their very constructive comments. This work was supported by NASA ROSES award NNX12AB72G to H.R., T.P., and W.C. and Danish Ministry of Science, Technology and Innovation award 11-115166 to W.C.

References


Bader H., and F. Small (1955), Sewage disposal at ice cap installations, Snow Ice Permafrost Research Establishment, Report 21.


Benson C. (1961), Stratigraphic studies in the snow and glacier ice of the Yukon Territory, Canada, J. Glaciol., 333–344.


Clason, C., et al. (2012), Modelling the delivery of supraglacial meltwater to the ice/bed interface: Application to southwest Devon Ice Cap, Nunavut, Canada, J. Glaciol., 58, 361–374.


Joughin, I., et al. (2010b), *MEaSUREs Greenland Ice Sheet Velocity Map from InSAR Data*, National Snow and Ice Data Center, Boulder, Colorado USA.


Seddik, H., et al. (2012), Simulations of the Greenland ice sheet 100 years into the future with the full Stokes model Elmer/Ice, *J. Glaciol.*, 58, 427–440.


